國立臺灣大學　　年度基礎學科免修認證考試試題

題號：2012

科目：微積分 2

題號：2012

※ 注意：請於試卷上「非選擇題作答區」標明題號並依序作答。

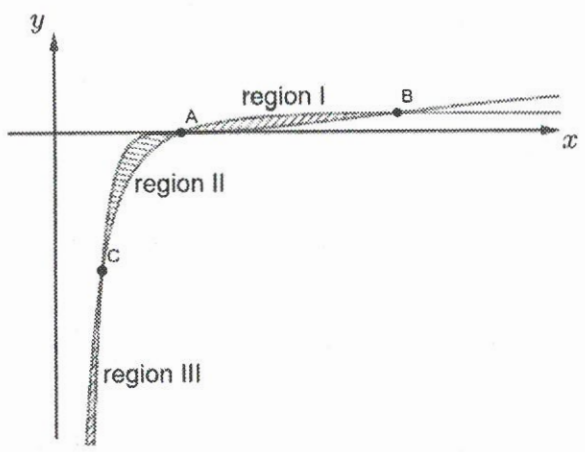
※ 禁止使用計算機

考試須知：

- ▶ 不能使用計算機，電子辭典及個人自備之計算紙。
- ▶ 無論計算或證明題，皆應詳述過程、理由；如未寫出詳細過程，一律不給分。
- ▶ 將答案寫於試卷，並標示正確的題號。

1. Evaluate the following integrals.
   (a) (10 pts) $\int \frac{dx}{x\sqrt{x^2+x+1}}$, $x>0$. (Hint : You may set $x=\frac{1}{t}$.)
   (b) (10 pts) $\int \frac{dx}{(\sin x)^4}$.
   (c) (10 pts) $\int \frac{3x^3-2x-2}{x^2(x^2+1)}dx$.

2. Consider two curves $y=\frac{\ln x}{x}$ and $y=\frac{(\ln x)^3}{x}$.
   (a) (10 pts) Find the area of the two regions (region I, II) enclosed by the curves.
   (b) (10 pts) Determine whether the region between the curves under the point C (region III) has finite area.



3. (10 pts) Let R be the region between the x-axis and $y=2\cos^2(x/4)$ for $0\le x\le 2\pi$. Find the volume of the solid generated when R is rotated about the y-axis.

4. A tank contains 100 L of water. A solution with a salt concentration of 0.4 kg/L is added at a rate of 5 L/min. The solution is kept mixed and is drained from the tank at a rate of K L/min.
   (a) (10 pts) When $K=5$, the amount of salt $y(t)$ satisfies $\frac{dy}{dt}=2-\frac{5y}{100}$. Solve the equation.
   (b) (10 pts) When $K=4$, the amount of salt $y(t)$ satisfies $\frac{dy}{dt}=2-\frac{4y}{100+t}$. Solve the equation.

5. C is a curve given by the polar equation $r=\sin\theta+\cos\theta$.
   (a) (5 pts) Convert the polar equation into an equation of x and y. Then sketch the curve C.
   (b) (15 pts) Find the area of the region in the fourth quadrant that is outside C and inside the curve $r^2=\cos(2\theta)$.

試題必須隨卷繳回