

考試須知：

- ▶ 不能使用計算機，電子辭典及個人自備之計算紙。
- ▶ 無論計算或證明題，皆應詳述過程、理由；如未寫出詳細過程，一律不給分。
- ▶ 將答案寫於試卷，並標示正確的題號。

1. (10%) Evaluate $\lim_{x \rightarrow 0} \frac{\int_x^{\tan x} e^{t^2} dt}{x^3}$.
2. (20%) Evaluate the following integrals.
 - (a) (5%) $\int (\ln x)^2 dx$.
 - (b) (5%) $\int \frac{1 + \sin x + \cos x}{e^x} dx$.
 - (c) (5%) $\int \frac{x^7 + 1}{x^3 + x} dx$.
 - (d) (5%) $\int_0^3 \frac{1}{\sqrt{4x - x^2}} dx$.
3. (10%) Find the value of the constant C for which the integral

$$\int_0^{\infty} \left(\frac{1}{\sqrt{x^2 + 1}} - \frac{C}{x + 1} \right) dx$$

converges. Evaluate the integral for this value of C .

4. (20%) (a) (10%) Let $P(t)$ be a function of time t , which satisfies

$$\frac{dP}{dt} = P(100 - P) \text{ and } P(0) = 1.$$

Find $P(t)$.

- (b) (10%) Solve $x^2 y' + 3xy = 1, y(1) = 1$ for $x > 0$.
5. (20%) Let R be the region enclosed by $y = 1 + x^2, y = 1 - x^2$ and $x = 1$.
 - (a) (10%) Find the perimeter(arc length around the shape) of R .
 - (b) (5%) Find the volume of the solid generated when R is rotated about the line $x = 2$.
 - (c) (5%) Find the volume of the solid generated when R is rotated about the line $y = -1$.
6. (20%) Consider the polar curve $r = (\sin \theta + \cos \theta)^{-2}$ in the first quadrant ($0 \leq \theta \leq \pi/2$).
 - (a) (10%) Find the Euclidean coordinates of the point on the polar curve that is closest to the origin.
 - (b) (10%) Find the area of the region bounded by the polar curve, the x -axis, and the y -axis.
(Hint: The formula for the area bounded by a polar curve is $\int_a^b \frac{1}{2} r^2 d\theta$.)

試題必須隨卷繳回