考試須知：

- ► 不能使用計算機，電子辭典及個人自備之計算紙。
- ► 無論計算或證明題，皆應詳述過程、理由；如未寫出詳細過程，一律不給分。
- ► 將答案寫於試卷，並標示正確的題號。

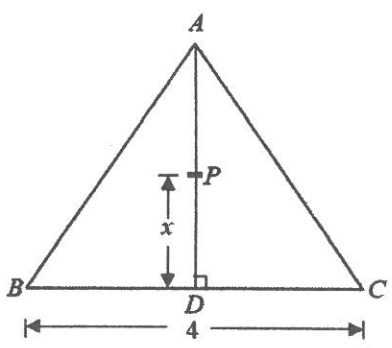
1. (20%) Compute the following limits or show that the limit doesn't exist.
   (a) (5%) $\lim_{x\to 0} \frac{x}{\sqrt{1-\cos 2x}}$.
   (b) (5%) $\lim_{x\to -\infty} \sqrt{x^2-x}\sin(\frac{1}{2x})$.
   (c) (10%) $\lim_{x\to 0^+} \left(\frac{a^x+b^x}{2}\right)^{\frac{1}{x}}$ where $a > b > 0$.

2. (20%) Compute derivatives.
   (a) (5%) $f(x) = \tan^{-1}(\sin^{-1}(\sqrt{x}))$. Find $f'(x)$.
   (b) (5%) $f(x) = \log_{x^2+1}|\sec x + \cos 2x|$. Find $f'(x)$.
   (c) (10%) $x + 2y + 1 = \frac{y^2}{x-1}$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $(2,-1)$.

3. (20%) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function on $\mathbb{R}$ with the following properties :
   (1) $\lim_{x\to 0} \frac{f(x)-1}{x} = 2020$.
   (2) $f(x+y) = f(x)f(y)$ for all real numbers x and y.
   (a) (5%) Find $f(0)$ and $f'(0)$.
   (b) (5%) Prove that $f(x)$ is differentiable on $\mathbb{R}$.
   (c) (10%) By considering the derivative of $f(x)\cdot e^{-2020x}$, determine the function $f(x)$.

4. (25%) Let $f(x) = \frac{\ln x}{x}$ for $x > 0$.
   (a) (5%) Find the interval of increase and interval of decrease of $f(x)$. Find the absolute maximum of $f(x)$.
   (b) (5%) Show that $\pi^e < e^\pi$.
   (c) (10%) Sketch the curve $y = f(x)$. Indicate on your sketch (if any) the local extrema, inflection points and asymptotes of the curve.
   (d) (5%) Show that for all $1 < b < e$ there is some $x > e$ such that $\frac{\ln x}{x} = \frac{\ln b}{b}$.

5. (15%) The figure below shows an isosceles triangle ABC with $AB = AC$ and $BC = 4$. D is the foot of perpendicular from A to BC and P is a point on AD with $AD = 3$. Let $PD = x$ and $r = PA + PB + PC$, where $0 \le x \le 3$. Find the greatest and least values for r as x varies.



試題必須隨卷繳回